This paper studies the nonlinear dynamic behaviors of a rigid rotor supported by ultra short gas bearing (USGB) system. A hybrid numerical method combining the differential transformation method and the finite difference method are used to calculate pressure distribution of USGB system and rotor orbits. The results obtained for the orbits of the rotor center are in good agreement with those obtained using the traditional finite difference approach. Moreover, the hybrid method avoids the numerical instability problem suffered by the finite difference scheme at low values of the rotor mass and computational time-step. The results presented summarize the changes which take place in the dynamic behavior of the USGB system as the bearing number are increased and therefore provide a useful guideline for the bearing system.

The pressure in the gas film is nonlinear, and the ratio of the film thickness to the molecular mean free path of the gas may be sufficiently low to prompt a slip flow effect; hence the bearing system performs non-periodic motion and causes micro structural fatigue to the system. So, in order to understand and control under what kind of operating condition the non-periodic motion will occur to the bearing system, a Reynolds’ equation, suitably modified to take account of the slip-flow effect, will be employed to identify the conditions under which non-periodic motion takes place. The Reynolds’ equation will be solved using a finite difference method and a novel hybrid method, respectively. The dynamic behavior of the rotor center will be examined under different operating conditions by generating the corresponding Poincaré maps, Lyapunov exponents etc. Based upon the numerical results, guidelines will be produced to specify suitable operating conditions which ensure periodic motion of a USGB system and therefore protect the microstructures containing such bearing systems from structural fatigue.

1. Introduction

Gas bearing systems have been extensively used for a variety of electro-mechanical system applications, and it is particularly valuable when used with precision instruments. This is due, in part, to low noise when there is rotation, and to zero friction when the instruments are used as null devices. This bearing system has a number of advantages compared to their rolling-element or oil-lubricated counterparts, including low friction losses and zero risk of contamination through lubricant leakage. As a result, they are widely applied in a diverse range of rotational systems.

Ultra short gas bearings (USGB) system is different from general gas bearings due to the limit of length-diameter ratio and ultra-thin lubrication. Because of the rotational speed of the rotor could reach 106 rpm, the stability of rotor dynamics is one of the most important factors for considering the design of USGB systems. So, how to increase the stability of rotor systems and control the appearance of non-periodic motion becomes the major execution of this paper.

The pressure in the gas film is nonlinear, and the ratio of the film thickness to the molecular mean free path of the gas may be sufficiently low to prompt a slip flow effect; hence the bearing system performs non-periodic motion and causes micro structural fatigue to the system. So, in order to understand and control under what kind of operating condition the non-periodic motion will occur to the bearing system, a Reynolds’ equation, suitably modified to take account of the slip-flow effect, will be employed to identify the conditions under which non-periodic motion takes place. The Reynolds’ equation will be solved using a finite difference method and a novel hybrid method, respectively. The dynamic behavior of the rotor center will be examined under different operating conditions by generating the corresponding Poincaré maps, Lyapunov exponents etc. Based upon the numerical results, guidelines will be produced to specify suitable operating conditions which ensure periodic motion of a USGB system and therefore protect the microstructures containing such bearing systems from structural fatigue.

According to the recent research, nonlinear dynamic responses of rotor-bearing systems are analyzed and published. In 1994, Malik and Bert [1] studied the differential quadrature method (DQM) and applied it for the first time to the solution of steady state oil and gas lubrication problems of self-acting hydrodynamic bearings. In that work, the quadrature solutions of the Reynolds equation for incompressible lubrication were compared with the exact solutions of finite-length bearings. The quadrature solutions of the compressible Reynolds equation for finite-length plain journal bearings were compared with the finite difference and finite element solutions. The
work also included comparison of the CPU times of the quadrature solutions with those of the trigonometric series and finite element solutions of oil-lubricated plain slider and journal bearings. For the gas-lubricated journal bearing, the CPU times of the quadrature solutions were compared with those of the finite solutions. In all cases it was found that the quadrature method was capable of yielding accurate solutions to the lubrication problems and that it was computationally more efficient than other methods of solution. Zhao et al. [2] investigated the sub-harmonic and quasi-periodic motions of an eccentric squeeze film damper-mounted rigid rotor system. The authors showed that for large values of the rotor unbalance and static misalignment, the sub-harmonic and quasi-periodic motions generated at speeds of more than twice the system critical speed bifurcated from an unstable harmonic solution. Sundararajan et al. [3] utilized a simple shooting scheme integrated with an arc-length continuation algorithm to analyze the dynamics of periodically-forced rotor systems. The results revealed the occurrence of periodic, quasi-periodic or chaotic motion at different values of the rotor speed. In 2001, Wang et al. [4] investigated static and dynamic characteristics of a flexible rotor supported by externally pressurized porous gas journal bearings. In this work, a modified Reynolds equation was solved by the finite difference method and a comparative stability of rotor center and journal center was done.

In 2006, Rahmatabadi et al.[5-6] studied the static and dynamic characteristics of noncircular gas journal bearings by considering the effect of mount angles and preload. They proved that noncircular bearings have better dynamic characteristic than circular bearings. Also, by using suitable value of mount angles stability margin can be increased. Although previous works provide insight into the behavior of the system but the bifurcation and nonlinear dynamic behavior of the gas film in a noncircular gas journal bearing has not examined. Therefore, this paper presents study of nonlinear dynamic behavior of a rigid rotor supported by two-lobe noncircular gas journal bearings.

Wang et al. [7-8] analyzed the bifurcation behavior and nonlinear dynamics of flexible and rigid rotors supported by gas journal bearings(relative short gas journal bearings, and relative short spherical gas bearing systems) and showed that the rotors exhibited a complex dynamic behavior comprising periodic, sub-harmonic, and quasi-periodic responses at different values of the rotor mass and bearing number, respectively. In 2009, Wang et al. [9] analyzed the bifurcation behavior of a spherical gas journal bearing system by a hybrid method including differential transformation method and finite difference method. The bearing system is modeled as a rigid rotor supported by bearing forces as a result of gas viscosity and rotational speed.

The present study analyzes the nonlinear dynamic response of a rigid rotor supported by two ultra short gas bearings. In analyzing the bearing system, the time-dependent motions of the rotor center are described using the Reynolds equation. The modified Reynolds equation is solved by using a hybrid method combining the differential transformation method (DTM) and the finite difference method (FDM). The validity of the hybrid method is confirmed by comparing the results obtained for the orbits of the rotor center with those obtained using the conventional FDM scheme. The proposed method is then applied to analyze the nonlinear dynamic response of the rotor for bearing number in the ranges 1.0 ~ 3.5, respectively.

2. Mathematical Modeling

2.1 Governing Equations

An ultra short gas bearing is different from general gas bearing system and the ratio of bearing length to diameter (L/D) is less than 0.1. The application for high rotational speed more than 106 rpm can be used by USGB system and the stability of this system is focused to be analyzed. The USGB system is shown in Fig. 1. O1 and OB are the center of rotor and bearing, respectively.

In general, the pressure distribution in the gas film between the shaft and the bushing in a ultra short gas bearing (USGB) system is modeled using the Reynolds equation for an ideal gas. In the present study, the time-dependent motions of the rotor center are modeled with:

\[ H \phi \left[ \frac{\partial^2 \phi}{\partial \theta^2} + \frac{D R}{C_r} \frac{\partial^2 \phi}{\partial \phi^2} + H \left( \frac{\partial^2 \phi}{\partial \phi^2} + \left( \frac{D R}{C_r} \right)^2 \frac{\partial^2 \phi}{\partial \theta^2} \right) \right] = 0 \]  

(1)

Where \( \phi = PH \), \( P \) is the dimensionless pressure corresponding to the atmospheric pressure \( Pa \), \( H \) is the dimensionless film thickness between the rotating shaft and the bushing, corresponding to the radial clearance \( Cr \), \( D \) is the diameter of bearing, \( L \) is the length of bearing, \( \lambda \) is the bearing number; \( \theta \) and \( \phi \) are the coordinates in the circumferential and axial directions, respectively.

The USGB system model incorporates the following design assumptions:

a. The gas flow in and out of the sides of the bearing (side flow) is neglected.

b. We assume that the flow is isothermal because the ability of the bearing materials to conduct away heat is greater than the heat generating capacity of the gas-film.

c. As gas viscosity is somewhat insensitive to changes in pressure and the temperature is virtually constant, we assume the gas viscosity to be constant.

The gas film pressure distribution must fulfill the following boundary conditions:

a. gas pressure on both ends of the housing is equal to the atmospheric pressure.

b. gas pressure \( P \) is an even function for \( \phi \).

c. gas pressure \( P \) is continuous at \( \phi = 0 \).

d. gas pressure \( P \) is a periodic function for \( \theta \).

The analysis presented in this study considers a USGB system comprising a perfectly-balanced rigid rotor of mass \( Mr \) supported symmetrically on two identical noncircular aerodynamic journal bearings, mounted in turn on rigid pedestals. Since the rotor is perfectly balanced and the NABS is symmetric about its central axes, the current analyses are confined to a single bearing supporting a
rotating rotor of mass $M_r$ with two degrees of translatory oscillation in the transverse plane.

In the transient state, the equation of motion of the rotor can be written as

$$ F_{ch} - F_{gh} = M_r \frac{d^2 X^*}{dt^2}, \quad F_{ch} - F_{gh} = M_r \frac{d^2 Y^*}{dt^2}, $$

where $F_{ch}$ and $F_{gh}$ are the components of the external loading in the horizontal and vertical directions, respectively.

2.2 Mathematical Formulation of Numerical Simulations

In solving the modified Reynolds equation using the finite difference method, (1) is discretized initially using the central-difference scheme in the $\theta$ and $\phi$ directions and is then discretized once again using the implicit-back-difference scheme in the time domain. Note that for simplicity, a uniform mesh size is used. Equation (1) can be transformed into the following form:

$$
\begin{align*}
(\Pi_{ij}^n) & = \left[ \frac{\phi_{i,j}^{n+1} - 2\phi_{i,j}^{n} + \phi_{i,j}^{n-1}}{(\Delta \theta)^2} + \frac{\phi_{i,j}^{n+1} - 2\phi_{i,j}^{n+1} + \phi_{i,j}^{n-1}}{(\Delta \phi)^2} \right] \\
(\Pi_{ij}^n) & = \left[ \frac{\phi_{i,j}^{n+1} - 2\phi_{i,j}^{n} + \phi_{i,j}^{n-1}}{(\Delta \theta)^2} \right] + \frac{\phi_{i,j}^{n+1} - \phi_{i,j}^{n}}{(\Delta \phi)} + 2A \frac{\phi_{i,j}^{n+1} - \phi_{i,j}^{n}}{\Delta \phi} \\
\phi_{i,j}^{n+1} & = \left[ \frac{\Pi_{ij}^n}{H} \right] + A \frac{\phi_{i,j}^{n+1} - \phi_{i,j}^{n}}{\Delta \phi} + 2A \frac{\phi_{i,j}^{n+1} - \phi_{i,j}^{n}}{\Delta \phi}
\end{align*}
$$

(3)

The pressure distribution at each time step can then be obtained using an iterative calculation process.

The hybrid method proposed in this study is commenced by using the differential transformation method (DTM) [9-10] to discretize the modified Reynolds equation given in (1) with respect to time. DTM is one of the most widely used methods for solving both linear and nonlinear differential equations due to its rapid convergence rate and minimal calculation error.

In solving the Reynolds equation for the current USGB system, DTM is used to transform the Reynolds equation with respect to the time domain, and thus (1) becomes

$$
\begin{align*}
H \otimes \phi \otimes \nabla^2 \phi + \left( \frac{D}{L} \right) \left[ \frac{D}{L} H \otimes \phi \otimes \nabla^2 \phi + \left( \frac{D}{L} \right)^2 H \otimes \phi \otimes \phi \right] + A \frac{\partial \phi}{\partial \theta} + \frac{\partial \phi}{\partial \phi} &= \frac{\partial \phi}{\partial \theta} + \frac{\partial \phi}{\partial \phi} + 2A \frac{\partial \phi}{\partial \phi}
\end{align*}
$$

\begin{align*}
\frac{\partial^2}{\partial \theta^2} + A \frac{\partial \phi}{\partial \theta} + \frac{\partial \phi}{\partial \phi} &= \frac{\partial \phi}{\partial \theta} + \frac{\partial \phi}{\partial \phi} + 2A \frac{\partial \phi}{\partial \phi}
\end{align*}

(4)

where

$$
S(k) = \phi^2 = \phi \otimes \phi = \sum_{i=0}^{k} \phi_{i,j}(k-l) \phi_{i,j}(l)
$$

(5)

The finite difference method (FDM) is then used to discretize (4) with respect to the $\theta$ and $\phi$ directions. Note that (4) is discretized using the second-order-accurate central-difference scheme for both the first and the second derivatives.

Substituting (5) into (4) yields

$$
\begin{align*}
\sum_{i=0}^{k-1} H_{i,j}(k-l) & \sum_{m=0}^{l-m} \left( \frac{\phi_{i,j}(m) - 2\phi_{i,j}(m) + \phi_{i,j}(m)}{(\Delta \theta)^2} \right) \\
& + \frac{D}{L} \sum_{i=0}^{k-1} H_{i,j}(k-l) \sum_{m=0}^{l-m} \left( \phi_{i,j}(l-m) - 2\phi_{i,j}(l-m) + \phi_{i,j}(l-m) \right) \left(\frac{\Delta \theta}{\Delta \phi}\right)^2 \\
& + \sum_{i=0}^{k-1} H_{i,j}(l-k) \sum_{m=0}^{l-m} \left( \phi_{i,j}(l-m) - 2\phi_{i,j}(l-m) + \phi_{i,j}(l-m) \right) \left(\frac{\Delta \theta}{\Delta \phi}\right)^2
\end{align*}
$$

(6)

where $\bar{H}$ is the step time value.

From (6), $P_{i,j}(k)$ is obtained for each time interval, where $i$ and $j$ are the coordinates of the node position and $k$ indicates the $k$th term.

The motions of the rotor center are computed using an iterative procedure which commences by determining the acceleration and then computes the velocity and the displacement, on a step-by-step basis over time. In defining the initial conditions, the initial displacement ($X_0, Y_0$) is specified as the static equilibrium position of the shaft and defines the gap $H_A(k)$ between the shaft and the journal bearing, and the velocity of the rotor is assumed to be zero.

The iterative computation procedure can be summarized as follows:

Step 1:
At time $\tau = 0$, the external loading increases from $F_{ch}$ to $F_{gh}$. After a time increment $\Delta \tau$, the new values of the rotor acceleration, velocity and displacement can be estimated.

Step 2:
The rotor displacement causes a change in the gap $H$ between the rotor and the bushing. Substituting the new value of $H$ into (1) gives the new pressure distribution in the gap $H$.

Step 3:
The internal force is estimated by integrating the pressure distribution obtained from Step 2.

Step 4:
The displacement and velocity values from Step 1, the pressure distribution from Step 2, and the internal force from Step 3 are taken as the new initial conditions. Using these new conditions, the calculation procedure returns to Step 1 to compute the changes which take place in the time interval $\Delta \tau \rightarrow 2\Delta \tau$ .

In this study, the data generated via the iterative computation procedure is then input into the FEM analysis to estimate the stress and strain in the system.
procedure described above are used to construct power spectra, Poincaré maps, bifurcation diagrams and Lyapunov exponents with which to examine the nonlinear dynamic response of the USGB system over representative ranges of bearing number. Note that in analyzing the behavior of the USGB system, the time-series data corresponding to the first 1000 revolutions are deliberately excluded in order to ensure that the analyzed data correspond to steady state conditions.

3. Results and Discussions

3.1 Numerical Analysis

Table 1 compares the results obtained from the FDM and hybrid method (FDM&DTM) for the orbits of the rotor center. It is observed that a good agreement exists between the two sets of results. Moreover, it can be seen that while the FDM suffers numerical instability at low values of time step, the hybrid method converges under all the considered conditions and therefore represents a more appropriate method for analyzing the nonlinear dynamic response of the USGB system.

Table 2 compare the Poincaré map data calculated by the hybrid method using different time step values, $H$, for bearing number values. For a given rotor mass and bearing number, the rotor center orbits are in agreement to approximately 5 decimal places for the different time steps, $H$.

3.2 Dynamic Analysis

The bearing number $\Lambda$ was increased over the range $1.0 \leq \Lambda \leq 3.5$.

3.2.1 Dynamic Orbits and Phase Trajectories

Fig. 2.1(a)-2.8(a) show that the dynamic orbits of the rotor center are regular at a low value of the bearing number ($\Lambda=1.8$), but become irregular at $\Lambda=2.59$. At a bearing number of $\Lambda=3.04$, 3.28 and 3.4, the rotor center resorts to a regular periodic motion. When the bearing number is changed to 3.08, 3.33 and 3.43, the rotor center performs an irregular motion. Fig. 2.1(b)-2.8(b) show the phase trajectories of the rotor center at different values of the bearing number. It is observed that the results are consistent with those shown in Fig. 2.1(a)-2.8(a) for the rotor center orbits, namely a regular motion at $\Lambda=1.8$, but a non-periodic motion at $\Lambda=2.59$, 3.04, 3.08, 3.28, 3.33, 3.4, and 3.43.

3.2.2 Power Spectra

Fig. 3.1(a)-3.8(a) and 3.1(b)-3.8(b) show that the rotor center performs a periodic motion at a bearing number of $\Lambda=1.8$. However, when the bearing number is increased to $\Lambda=2.59$, the power spectra (Fig. 3.2(a)-3.2(b)) show that the rotor center performs quasi-periodic motion in both the horizontal and the vertical directions. For values of $\Lambda$ equal to 3.04, 3.08, 3.28, 3.33, 3.4, and 3.43, the rotor center performs non-periodic motion.

3.2.3 Bifurcation Diagrams, Poincaré Maps and Maximum Lyapunov exponents

Fig. 4 plots the bifurcation diagrams for the rotor center displacement in the horizontal and vertical directions as a function of the bearing number $\Lambda$ in the range 1.0 to 3.5. Fig. 5(a)-5(b) present the Poincaré maps of the rotor center trajectories at $\Lambda=1.8$, 2.59, 3.04, 3.08, 3.28, 3.33, 3.4, and 3.43, respectively. In Fig. 4, it can be seen that the rotor center performs T-periodic motion over the bearing number range $1.0 \leq \Lambda \leq 2.59$ and is replaced by quasi-periodic motion (see Fig. 5(a)). This quasi-periodic behavior persists over $2.59 \leq \Lambda \leq 3.04$, and also changes to sub-harmonic with a period of multi-T. For bearing numbers in the range $3.04 \leq \Lambda \leq 3.5$, the rotor center transits through Multi-T→Chaotic→Multi-T→Chaotic→Multi-T→Chaotic motion (see Figs. 5(c)-5(h)). Figure 6 shows that the maximum Lyapunov exponent has a positive value when $\Lambda$ equals to 3.08, 3.33 and 3.43, and also indicates that the system has a chaotic response. Meanwhile, over the intervals $3.08 \leq \Lambda \leq 3.28$, $3.33 \leq \Lambda \leq 3.4$, and $3.43 \leq \Lambda \leq 3.5$, shown in Figs. 4 and 6, the system behaves non-stable behavior and also should be avoided to operate under these parameters.

In other words, these bearing number intervals, which correspond to the typical operating conditions of a real-world USGB system, are characterized by T-, quasi-periodic, multi-T and chaotic motions of the rotor center. However, at specific and higher bearing numbers, the rotor performs predominantly multi-T-sub-harmonic and chaotic-periodic motions in the horizontal and vertical directions. Table 3 summarizes the motions performed by the rotor center for bearing number values in the interval $1.0 \leq \Lambda \leq 3.5$.

4. Conclusions

This study has analyzed the nonlinear dynamic behavior of an ultra short gas bearing (USGB) system by utilizing a hybrid numerical scheme comprising the differential transformation method (DTM) and the finite difference method (FDM). The system dynamic orbits, phase trajectories, power spectra, bifurcation diagrams, Poincaré maps, and maximum Lyapunov exponents have revealed the presence of a complex dynamic behavior comprising periodic, sub-harmonic, quasi-periodic and chaotic responses of the rotor center. Although, this hybrid method might be not the best candidate, but from Table 1 the results obtained by the FDM and hybrid method (DTM&FDM) for the orbits of the rotor center prove that a good agreement exists between different sets of results. Moreover, the DTM&FDM method converges under all the considered conditions and therefore represents a more appropriate method for analyzing the nonlinear dynamic response of the USGB system. The results of this study provide an understanding of the nonlinear dynamic behavior of ultra short gas bearing systems characterized by different bearing numbers $\Lambda$. Specifically, the results have shown that at $\Lambda=1.8$, the
Poincaré map contains one discrete points, indicating the presence of T-periodic motion. However, when the bearing number is increased to \( \Lambda = 2.59 \), the T-periodic motion is replaced by quasi-periodic motion. Then, chaotic motion appears in three intervals in the bearing number range \( 3.08 \leq \Lambda \leq 3.5 \). As shown in Table 3, the intermediate regions in the bearing number range are characterized by multi-T- or chaotic motion.

### Table 1 Comparison of rotor center orbits calculated by FDM and hybrid method, respectively. ( \( M_r = 3.0 \text{kg}, \omega = 2 \times 10^6 \text{rpm} \))

<table>
<thead>
<tr>
<th>Orbits</th>
<th>Method</th>
<th>X(nT)</th>
<th>Y(nT)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( H = 1 \times 10^{-3} )</td>
<td>( H = 1 \times 10^{-2} )</td>
<td>( H = 1 \times 10^{-3} )</td>
</tr>
<tr>
<td>FDM</td>
<td>-0.0802178</td>
<td>-0.0896257</td>
<td>0.0832310</td>
</tr>
<tr>
<td>Hybrid method</td>
<td>0.0822852</td>
<td>-0.0822361</td>
<td>0.0883342</td>
</tr>
</tbody>
</table>

### Table 2 Comparison of Poincaré maps of rotor center with different values of \( \Lambda \) and \( h \) by Hybrid method.

<table>
<thead>
<tr>
<th>( \Lambda )</th>
<th>( \tau )</th>
<th>X(nT)</th>
<th>Y(nT)</th>
<th>( \tau )</th>
<th>X(nT)</th>
<th>Y(nT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8</td>
<td>( \pi /300 )</td>
<td>0.0102304</td>
<td>-0.0304204</td>
<td>( \pi /300 )</td>
<td>-0.2243914</td>
<td>-0.2183472</td>
</tr>
<tr>
<td></td>
<td>( \pi /600 )</td>
<td>0.0102381</td>
<td>-0.0304262</td>
<td>( \pi /600 )</td>
<td>-0.2243935</td>
<td>-0.2183450</td>
</tr>
<tr>
<td>2.13</td>
<td>( \tau )</td>
<td>X(nT)</td>
<td>Y(nT)</td>
<td>( \tau )</td>
<td>X(nT)</td>
<td>Y(nT)</td>
</tr>
<tr>
<td></td>
<td>( \pi /300 )</td>
<td>0.0102304</td>
<td>-0.0304204</td>
<td>( \pi /300 )</td>
<td>-0.2243914</td>
<td>-0.2183472</td>
</tr>
<tr>
<td></td>
<td>( \pi /600 )</td>
<td>0.0102381</td>
<td>-0.0304262</td>
<td>( \pi /600 )</td>
<td>-0.2243935</td>
<td>-0.2183450</td>
</tr>
</tbody>
</table>

### Table 3. Behavior of rotor center at different bearing numbers in interval \( 1.0 \leq \Lambda \leq 3.5 \).

<table>
<thead>
<tr>
<th>Bearing number ( \Lambda )</th>
<th>Dynamic Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1.0 \leq \Lambda &lt; 2.59 )</td>
<td>T Quasi Multi-T Chaos</td>
</tr>
<tr>
<td>( 2.59 \leq \Lambda &lt; 3.04 )</td>
<td>Quasi Multi-T Chaos</td>
</tr>
<tr>
<td>( 3.04 \leq \Lambda &lt; 3.08 )</td>
<td>Multi-T Chaos Multi-T Chaos</td>
</tr>
<tr>
<td>( 3.08 \leq \Lambda &lt; 3.28 )</td>
<td>Multi-T Chaos</td>
</tr>
<tr>
<td>( 3.28 \leq \Lambda &lt; 3.33 )</td>
<td>Multi-T Chaos</td>
</tr>
<tr>
<td>( 3.33 \leq \Lambda &lt; 3.4 )</td>
<td>Multi-T Chaos</td>
</tr>
<tr>
<td>( 3.4 \leq \Lambda &lt; 3.43 )</td>
<td>Multi-T Chaos</td>
</tr>
<tr>
<td>( 3.43 \leq \Lambda &lt; 3.5 )</td>
<td>Multi-T Chaos</td>
</tr>
</tbody>
</table>

Fig. 1 Cross-section of ultra short gas bearing system
Fig. 2 Trajectories of rotor center at $\Lambda = 1.8, 2.59, 3.04, 3.08, 3.28, 3.33, 3.4, 3.43$ (Figures 2.1(a)-2.8(a)); phase trajectories of rotor center (Figures 2.1(b)-2.8(b)) at $M = 2.5$ kg.

Fig. 3 Power spectra of rotor center in horizontal direction (Figures 3.1(a)-3.8(a)) and vertical direction (Figures 3.1(b)-3.8(b)) at $\Lambda = 1.8, 2.59, 3.04, 3.08, 3.28, 3.33, 3.4, 3.43$ (at $M = 2.5$ kg).

Fig. 4 Bifurcation diagrams: (a) $X(nT)$ and (b) $Y(nT)$ versus bearing number $\Lambda$ over interval $1.0 \leq \Lambda \leq 3.5$. 

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Fig. 5 Poincaré maps of rotor center trajectory at (a) $\Lambda=1.8$, (b) 2.59, (c) 3.04 (d) 3.08, (e) 3.28 (f) 3.33, (g) 3.4 (h) 3.43.

Fig. 6 Maximum Lyapunov exponents of system at different values of bearing number at: (a) $\Lambda=3.08$ (b) 3.33 (c) 3.43.

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